MILLIMETER WAVE GENERATION
BY CERENKOV RADIATION

M. Danos and H. Lashinsky
Columbia Radiation Laboratory
Columbia University
New York 27, N. Y.

As a part of the Columbia University Radiation Laboratory program for the generation of millimeter waves the Cerenkov effect is being used to produce microwave radiation. The frequency range of interest in this project is in the submillimeter region in as much as the longer wavelengths can be generated by conventional methods. I shall first describe briefly the properties of Cerenkov radiation and then report on results obtained to date and on the program planned for the immediate future.

An electron moving through a refractive medium with a velocity greater then the phase velocity of light in that medium emits radiation with a continuous spectrum. The emitted radiation is very weak[1] and because it is proportional to $1/\lambda$ it is much too small in the microwave region to be useful even when high current electron beams are employed. If one uses a bunched electron beam not only is the radiated power greatly increased but the radiated spectrum now is a line spectrum consisting of the bunching frequency and its harmonics. It is actually not necessary that the electrons pass through the medium. It is sufficient that the beam be located at a small distance from the dielectric. The results of a calculation for a two dimensional flat geometry as shown in Figure 1 are as follows: Let the charge density in a beam be given by

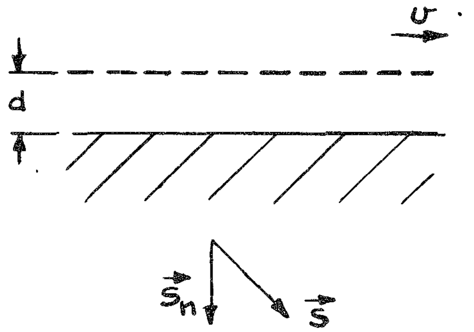

Figure 1. Flat beam over dielectric.

$$\rho=\rho_0\left[1+\alpha\cos\left(\frac{\omega}{v}z-\omega t\right)\right]\delta(x-x_0) \qquad (1)$$

Here $\delta(x-x_0)$ is Dirac's δ -function, ω the bunching frequency, v the velocity of the electrons, and α the Fourier component of the density of the frequency . Expression (1) therefore describes a thin, flat beam located at $x=x_0$ and bunched with a relative amplitude α. If the bunches are very short compared to v/ω, i.e. optimum bunching, then $\alpha=2$. Let the dielectric constant of the medium be ϵ . Then the time average of the component of the Poynting vector normal to the beam, S_n , is given by:

$$S_n = S_0\,(I\alpha)^2\,e^{-2qd/\lambda}\,\eta(\epsilon,\beta) \qquad (2)$$

Here the free space wavelength is $\lambda=2\pi c/\omega$, $\beta=v/c$

$$q=\frac{2\pi}{\beta}\sqrt{1-\beta^2} \qquad (3)$$

$$\eta(\epsilon,\beta)=\frac{\epsilon}{\beta}\,\frac{(1-\beta^2)\sqrt{\epsilon\beta^2-1}}{\epsilon^2(1-\beta^2)+(\epsilon\beta^2-1)} \qquad (4)$$

and

$$S_0=\frac{2\pi e^2}{c}N^2 \qquad (5)$$

where N is the number of electrons in the unit charge. If I is expressed in ma/cm, the numerical value of S_0 is

$$S_0 = 189\,\mu W/cm^2 \qquad (5')$$

Let us examine expression (2). The factor exp $(-2qd/\lambda)$ is universal for any arrangement in which the ac-energy contained in an electron beam is extracted. It describes the well known difficulty in millimeter tubes, where the distance between beam and structure has to be kept very small compared to the wavelength. It may be mentioned that for 10 Kev electrons $2q \approx 60$. The factor $(I\alpha)^2$ is just the ac-component of the beam. Therefore the factor $S_0\eta$ can be interpreted as the intrinsic interaction impedance between the beam and radiation field; in other words, the radiative resistance for Cerenkov radiation. For the TiO_2 the dielectric constant $\epsilon = 100$, so that $S_0\eta \approx 15\Omega$. The conclusion one has to draw from this is that it does not pay to use Cerenkov radiation for microwave generation at wavelengths where other structures are possible, such as helixes etc. Only where the difficulties in building such structures are very great as in submillimeter region, may the Cerenkov radiation start to become advantageous. There is no difficulty in obtaining suitable materials for the dielectric. For example, TiO_2 is known to maintain its dielectric properties up to the long wavelength infrared.

An exploratory experiment has been carried out with an apparatus which is shown schematically in Figure 2. A flat electron beam is formed and passes through a klystrontype bunching cavity. After an appropriate drifting time ,

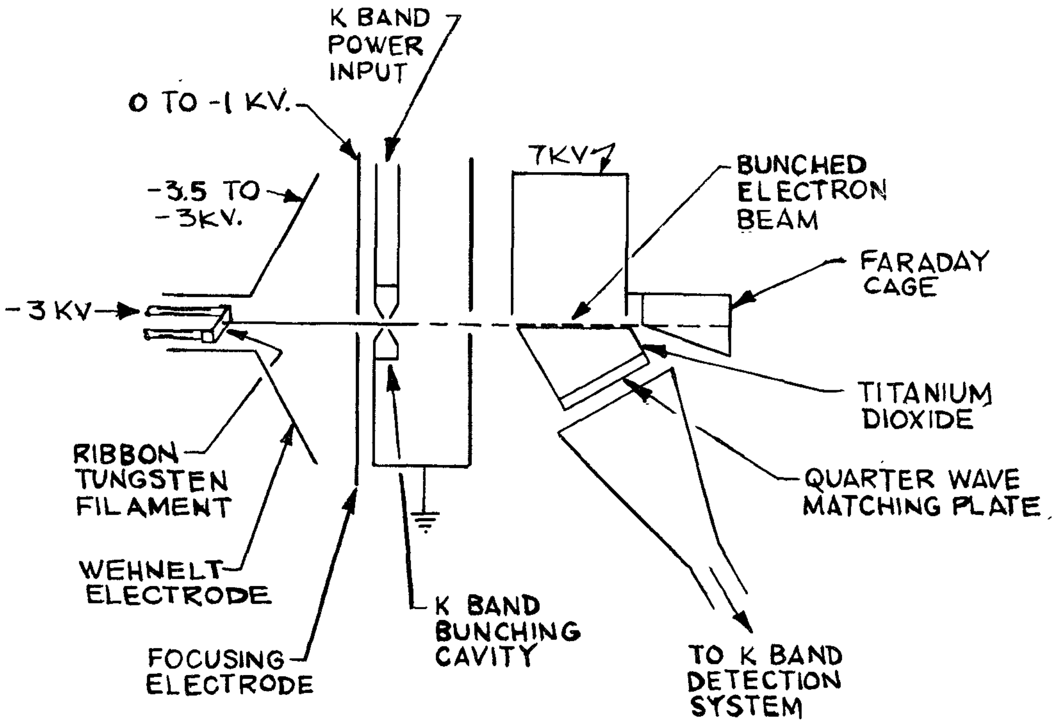

Figure 2. Experimental arrangement.

density bunches are formed and the beam passes over the dielectric at a close distance. The emitted radiation passes through the quarter wave matching plate and is collected by a horn. With this set-up radiation of roughly 10^{-7} W was obtained. The beam current was 0.4mA, the beam width 4mm, the length over the dielectric 1.7cm; the klystron was squarewave modulated at 6 kc/s; and the beam was switched on and off at a rate of 20 c/s. The 6kc modulation was employed to enable the use of a narrow band 6 kc amplifier system; by switching the beam on and off it was possible to distinguish between leakage and radiation associated with the electron beam. Inserting in (2) the value $d = 0.3$ mm and the other values as given above, one obtains a theoretical power output of the same order of magnitude.

It seems to be clear that it will not be possible to increase the power output greatly without increasing the current density of the electron beam appreciably. In order to do that, magnetic focussing will have to be employed. Therefore a new experiment is planned with the following general features:

a. focussing of the beam by a homogeneous magnetic field of the order of 10^4 gauss,

b. current density of the beam 10 A/cm^2 and

c. beam dimensions of 3mm width and 0.01 mm thickness, giving a beam current of 30 mA.

With a bunching frequency corresponding to 1 cm radiation, it is hoped that a power of 10^{-7} W can be obtained for $\lambda < 1$ mm. The main problem here, besides alignment difficulties, is the sharpness of the bunches. The estimate of 10^{-7} W has been obtained under the assumption that the harmonics contained in the beam decrease in power by a factor of 3 per harmonic. This assumption is based on the experimental observation of magnetron harmonics[2] where the radiated harmonic power of a magnetron depends on the product of the harmonic content of the electron stream and the efficiency of coupling between the electron stream and the structure at the harmonic frequency. Since the coupling efficiency decreases with frequency by a factor analagous to the exponential in (2), the above mentioned experimental result means that bunching in the electron stream in the magnetron is such that the harmonics do not decrease by more than a factor of three per harmonic. In any case, the theoretical harmonic content of a bunched beam, neglecting space charge effects is much higher than the above estimate and on this basis one would expect a much greater power output at submillimeter frequencies than the value quoted above.

In conclusion, it seems reasonable to expect that the brute force method outlined above will yield at least enough submillimeter power for the purposes of spectroscopy.

References

1. I.Tamm, J. Phys. USSR, 1, 439 (1939)
2. A. H. Nethercot, preceeding paper.